Exercise 35

Find the derivative of the function.

$$y = \cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned}
y' = \frac{dy}{dx} &= \frac{d}{dx}\left[\cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\right] \\
&= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{d}{dx}\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \\
&= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{\left[\frac{d}{dx}(1-e^{2x})\right](1+e^{2x}) - \left[\frac{d}{dx}(1+e^{2x})\right](1-e^{2x})}{(1+e^{2x})^2} \\
&= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{\left[(-e^{2x})\cdot\frac{d}{dx}(2x)\right](1+e^{2x}) - \left[(e^{2x})\cdot\frac{d}{dx}(2x)\right](1-e^{2x})}{(1+e^{2x})^2} \\
&= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{\left[(-e^{2x})\cdot(2)\right](1+e^{2x}) - \left[(e^{2x})\cdot(2)\right](1-e^{2x})}{(1+e^{2x})^2} \\
&= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{-2e^{2x}[(1+e^{2x})+(1-e^{2x})]}{(1+e^{2x})^2} \\
&= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{-2e^{2x}(2)}{(1+e^{2x})^2} \\
&= \frac{4e^{2x}}{(1+e^{2x})^2}\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)
\end{aligned}$$